Traffic Source Modeling

Helmut Hlavacs, Gabriele Kotsis, Christine Steinkellner

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Institute of Applied Computer Science and Information Systems
University of Vienna
Lenaugasse 2/8, A-1080 Wien, Österreich
[hlavacs|gabi]@ani.univie.ac.at

Abstract— **Designing and planning networks is often done by simulating the influence of various traffic types. This simulation approach depends on reliable and realistic traffic models that are capable of covering first- and second-order statistics of the observed network traffic.**

In this report, an overview over state-of-the-art models for the simulation of network traffic will be given.

Contents

I Introduction 1

II Modeling Network Traffic 1

III Renewal Models 2
- III-A Poisson Processes . . . 2
- III-B Bernoulli Processes . . . 2
- III-C Possible Applications . . . 2

IV Markov Models 2
- IV-A Markov Modulated Traffic models . . . 3
- IV-B Markov Modulated Poisson Processes . . . 3
- IV-C Generalizations of Markov Processes . . . 3

V Fluid Models 3

VI Linear Stochastic Models 3
- VI-A The DAR(p) Model . . . 3

VII TES 4

VIII Self-Similar Traffic Models 4
- VIII-A Fractional Brownian Motion . . . 6
- VIII-B Fractional Gaussian Noise . . . 6
- VIII-C ARFIMA . . . 6
- VIII-D Wavelets . . . 7
- VIII-E On/Off Processes . . . 8
- VIII-F Poisson-Zeta Process . . . 8
- VIII-G Deterministic Chaotic Maps . . . 8
- VIII-H Self-Similarity Through Aggregation . . . 9
- VIII-I The $M/G/\infty$ Model . . . 9
- VIII-J Superimposing AR(1) Processes . . . 9
- VIII-K Self-Similar Markov Modulated . . . 9
- VIII-L The GBAR and GBMA Processes . . . 9
- VIII-M Spatial Renewal Processes . . . 9
- VIII-N Multifractal Traffic . . . 10

IX Overview of Traffic Generators 10
- IX-A VBR Video Traffic / MPEG . . . 10
- IX-B Ethernet Traffic . . . 10
- IX-C ATM . . . 10
- IX-D Wan, TCP, Telnet, FTP . . . 11
- IX-E Web Traffic . . . 11

I. Introduction

Computer and telecommunications networks have become the basis of our economic and scientific infrastructure. Though network speed is growing faster and faster, sending data from one computer or terminal to the other is still being regarded as a bottleneck. Thus, when installing or upgrading large networks, thorough planning is of utmost importance. Planning the capacity of networks can be done by using analytical means, or by using simulation. Analytical models often impose restrictions to the modeled traffic that are not met in reality. On the other hand, when simulating network traffic, no such restrictions are necessary.

The aim of this report is to give an overview over the state of the art models for simulating network traffic.

II. Modeling Network Traffic

When generating artificial network traffic, streams of requests can occur on several different levels of description. Such a stream S of request in essence is characterized by a sequence of observations

$$\ldots,\, X(t_{n-1}),\, X(t_n),\, X(t_{n+1}),\, \ldots$$

at time points

$$\ldots,\, t_{n-1},\, t_n,\, t_{n+1},\, \ldots$$

These observations now can describe, for instance, inter-arrival times between successive user commands at the user behavior level or the inter-arrival times or sizes of data packets at the application or network level. Usually, the $X(t_i)$ are modeled by a family of random variables with known probability distribution function and time index t. If the set of possible values (the state space) is finite or countable, then the process is called discrete-state process, and a continuous-state process otherwise. The time index

t may be finite or countable, yielding a discrete-time process $S = \{X_n\}_{n=0}^{\infty}$ or may take any value in a set of finite or infinite intervals, yielding a continuous-time process $S = \{X_t\}_{t=0}^{\infty}$. If the process describes the arrival of single discrete entities (packets, cells, commands,...), it is called point process , consisting of a sequence of arrival instants $T_0 = 0, T_1, \ldots, T_n, \ldots$ measured from the origin 0. An alternative description is given by counting processes $\{N(t)\}_{t=0}^{\infty}$, a continuous-time, non-negative integer valued stochastic process, where $N(t) = \max\{n : T_n \leq t\}$ is the number of (traffic) arrivals in the interval $(0, t]$. Yet another description of point processes is given by inter-arrival time processes $\{A_n\}_{n=1}^{\infty}$, where $A_n = T_n - T_{n-1}$ is the length of the time interval separating the n-th arrival from the previous one. The equivalence of these descriptions follows from the fact that $T_n = \sum_{k=1}^{n} A_k$, and from the equality of events ([1])

$$\{N(t) = n\} = \{T_n \leq t < T_{n+1}\} =$$

$$= \left\{\sum_{k=1}^{n} A_k \leq t < \sum_{k=1}^{n+1} A_k\right\}.$$

III. Renewal Models

In a renewal process, the $X(t)$ are independent, identically distributed, but their distribution function is allowed to be general. Being independent means that the observation at time t does not depend on any observation in the past or future, thus the auto-correlation function for all lags $k \neq 0$ is equal to zero.

A. Poisson Processes

A Poisson process describes the arrival of observations at certain points in time. The n-th inter-arrival time A_n is described by an exponential distribution

$$P\{A_n \leq \tau\} = 1 - e^{-\lambda\tau}$$

with mean arrival rate (mean number of arrivals per time unit) λ. The number of arrivals within an interval of length τ is described by a counting process satisfying

$$P\{N(\tau) = n\} = \frac{(\lambda\tau)^n e^{-\lambda\tau}}{n!}.$$

Creating a Poisson model means identifying the correct mean arrival rate λ.

B. Bernoulli Processes

Bernoulli processes are the discrete-time analog of Poisson processes. Arrivals can only take place at some time slot k. The probability of an arrival in such time slot is p, independent of others. The number of arrivals for slot k is binomially distributed

$$P\{N_k = n\} = \binom{k}{n} p^n (1-p)^{k-n}.$$

The number of time-slots in between two arrivals is geometrically distributed with parameter p

$$P\{A_n = j\} = p(1-p)^j,$$

j being a non-negative integer.

Creating Bernoulli processes means estimating the correct arrival probability p.

C. Possible Applications

Renewal processes can be used to model arrivals, which are strictly independent from each other:

- The arrival of users to some company/computer facility.
- The arrival of network traffic packets, if the observed network traffic shows no auto-correlation.
- A stream of commands issued to an application, if no interdependencies on past results are observed.

In contradiction to the independence assumption, the observed traffic is often highly correlated ([2]). Assuming uncorrelated traffic might result in unrealistic models ([3]).

IV. Markov Models

A first step to describe dependencies between the $X(t)$ is given by Markov processes. A Markov process with discrete state space is called Markov chain ([4]). A set of random variables $\{X_n\}$ is called discrete-time Markov chain, if the probability that the next observed value (state) will be $x_{n+1} = j$ depends only on the current state $x_n = i$ and is given by p_{ij}. The dependency thus reaches back one unit in time and is also independent of the time, the process has spent in its current state (memoryless property). In the discrete-time case, the time spent in a particular state must therefore be geometrically distributed. For a continuous-time Markov chain, the state changes can occur at any time t and the memoryless property demands that the time spent in a particular state i is distributed exponentially with parameter λ_i (depending on the current state i).

Creating Markov Chains

At first, the set of possible states must be defined. In the discrete-time case, the transition probability matrix will then define the p_{ij}.

Possible Applications

Markov processes can be used to model processes, where the observations depend on only the previous observed value:

- User behavior: The next action is determined by the previous action plus maybe some return value (success, failure, ...)
- System/Network state change/failure
- Network traffic, if the observed traffic shows no or little auto-correlation.

If network traffic is to be generated by a continuous-time Markov chain, each transition from one state to the other (or possibly back to the same state) might represent an entity arrival ([1]). If traffic is generated with discrete time Markov chains, each state i can correspond to i idle slots separating successive slots, and is the probability of a j-slot separation, given that the previous one was an i-slot separation.

In [5], a Markov chain approach is taken to model the length of Groups Of Pictures (GOPs) in an MPEG-1 encoded video. The GOP length is split into 2 time series, one being the original sequence and one being smoothened by a moving average filter of length W. Both time series are then quantized into M and N steps, yielding two discrete state time series. Combining these two chains into one yields a discrete time discrete state Markov chain with associated probability transition matrix.

A. Markov Modulated Traffic models

In Markov-modulated models an explicit notion of state is introduced into the description of a traffic stream. An auxiliary Markov process is evolving in time and its current state controls the probability law of the traffic mechanism.

B. Markov Modulated Poisson Processes

In this scheme, the current state k of a superimposed Markov chain defines the currently used arrival rate λ_k of the modulated Poisson process. Each state may be assigned its own arrival rate λ_k

Creating MMPPs

Normally, the observed traffic (arrival rates) is quantized into n states, each being assigned a state in the Markov chain. Then, the appropriate Markov chain is constructed to drive the process.

Possible Applications

This model is applicable, if the observed arrival rates vary over time, yet, no significant correlation is measured.

C. Generalizations of Markov Processes

A further generalization of Markov processes leads to

- Phase-type renewal processes ([6]): The time until the next entity arrival is given by the time, an underlying continuous-time Markov chain needs to reach the absorbing state.
- Markov renewal processes ([1]): The distribution yielding the time of the next state change of a Markov chain is general and depends only on the current state.
- (Batch) Markovian Arrival Processes ([7],[8]): Similar to phase-type renewal processes.
- Discrete-Time (Batch) Markovian Arrival Processes ([9]): The discrete-time version of MAPs.

V. Fluid Models

If the number of arriving entities grows very large, each individual entity will add only negligible information to the traffic stream, just like the molecules in a water pipeline. As an example, the number of ATM cells per time unit might grow very large when sending high quality video information. When simulating such streams, the time granularity would be quite fine, and consequently simulation of all ATM cell arrivals would consume vast amounts of CPU time and main memory. A fluid simulation would assume that the incoming fluid flow remains (roughly) constant over much longer time periods. Traffic fluctuations would be modeled by events signalling a change of flow rate ([1],[10], [11]).

Possible Applications of Fluid Models are the following :

- ATM cells from near constant sources.
- TCP/IP traffic, if large volumes of data are transferred, and network conditions do no change.

VI. Linear Stochastic Models

The Markov property demands that the next observed state of a process can only depend on the current one, summing the whole process history into the current state. Autoregressive models define the next random variable in the sequence X_n as an explicit function of previous ones within a time window stretching from present to past. The distribution of the X_n is called marginal distribution, the autocorrelation function $\rho : N \to R$ yields for each $k \in N$, called the lag, the correlation coefficient of X_n and X_{n-k}. Autoregressive models are suitable for modeling short-range dependencies, but fail to model long-range dependencies as often measured in VBR-coded video, web and Ethernet traffic ([2]).

A discrete-time stochastic process $\{X_n\}_{n=0}^{\infty}$ is called Gaussian process, if for any finite set of time points $\{t_1, t_2, \ldots, t_n\}$, the corresponding random variables $\{X_{t_i}\}_{i=1}^{n}$ define a multivariate normal distribution. The marginal distribution thus consists of a normal distribution.

The class of linear stochastic models ([12]) has the form

$$X_n = a_0 + \sum_{r=1}^{\infty} (\alpha_r X_{n-r} - \beta_r \varepsilon_{n-r}) + \varepsilon_n,\ n > 0,$$

where the X_n are a family random variables, the α_r and β_r are real constants, the ε_n are zero-mean, iid random variables, called residuals or innovations, which are independent of the X_n. The most popular classes of linear stochastic models are called AR(p), MA(q), ARMA(p,q) and ARIMA(p,d,q). The ARFIMA(p,d,q) models use a similar scheme, but are designed to yield fractal (self-similar) output.

A. The DAR(p) Model

In [13], a special autoregressive process, called DAR(p) (discrete autoregressive process of order p), is used to simulate VBR traffic and to measure the effectiveness compared to self-similar models. Let $\{\varepsilon_n\}$ be a sequence of iid random variables taking values in Z, the set of integers, with distribution π. Let $\{V_n\}$ be a sequence of Bernoulli random variables with $P\{V_n = 1\} = 1 - P\{V_n = 0\} = \rho$ for $0 \le p < 1$. For the DAR(p) process, ρ represents the first-lag autocorrelation. Let $\{A_n\}$ be a sequence of iid random variables taking values in $\{1, \ldots, p\}$ with $P\{A_n = i\} = a_i \ge 0$, $i = 1, 2, \ldots, p$ and $\sum_{i=1}^{p} a_i = 1$. Let $S_n = V_n S_{n-A_n} + (1 - V_n)\,\varepsilon_n$ for $n \ge 1$, then the process $S = \{S_n\}$ is called DAR(p) process. This process has p degrees of freedom and can match up to the first p autocorrelations. It depends explicitly on the last p values. In

[13], it is also claimed that instead of taking into account all autocorrelations up to infinity, it is enough to take into account only a finite number up to an index called CTS (Critical Time Scale).

Creating Linear Stochastic Models

- First, the exact type of model must be identified, i.e. whether it is AR(p), MA(q), ARMA(p,q) or ARIMA(p,d,q) ([12]).
- Then, the parameters p, q and d must be identified. Usually, p and q are smaller than 2.
- Finally, the parameters α_i and β_j must be estimated ([12]). This can be done, for example, by using least-squares approximation ([14]).

Possible Applications

Because of their simplicity, AR(p) models are particularly well suited to model short-range dependencies:

- VBR coded video: Such videos produce a stream of frames of similar length, which can be modeled by autoregressive type models, while scene changes, causing a major burst, might be modeled by some modulating mechanism such as a Markov chain. In [15], the mixture of two AR(1) traffic models is used to generate VBR coded video.
- Network traffic with rapidly decaying autocorrelation function. Though linear stochastic models are members of the class of Gaussian models, the observed marginal distributions often differ from perfect Gaussian distributions by some skew.

VII. TES

Gaussian models assume Gaussian marginal distributions, yet real traffic measurements have revealed that this is not necessarily the case. More specifically, the observed marginal distributions often have heavier tails than Gaussian random variables. TES models ([16]) capture both marginals and autocorrelations of empirical records. The method assumes that time series (such as traffic measurements over time) are available. It aims to construct a model capturing the empirical marginal distribution (by using histograms), the leading autocorrelations up to a reasonable lag and yielding output that resembles the observed records. TES is based on the following principles:

1. The Inversion Method: Let F be any distribution function and $U \sim Uniform\,(0,1)$. Then the random variable $X \sim F^{-1}\,(U)$ satisfies $X \sim F$. The sequence $\{U_n\}$ with uniform marginal distribution is thus transformed into a sequence $\{X_n\}$ with marginal distribution F. This principle can be expanded by using the empirical histogram of observed values instead of F.
2. Modulo-1 Arithmetic: Let x be any real number, then the floor operator $\lfloor . \rfloor$ is defined by $\lfloor x \rfloor = \max\{n : n \leq x \wedge n \in Z\}$. The modulo-1 operator $\langle . \rangle$ is defined for any real x by $\langle x \rangle = x - \lfloor x \rfloor$.
3. Iterated Uniformity: Let $U \sim Uniform\,(0,1)$ and let V be any real random variable. Define $W = \langle U + V \rangle$. Then $W \sim Uniform\,(0,1)$. Furthermore, let $U_0 \sim Uniform\,(0,1)$, and $\{V_n\}_{n=1}^{\infty}$ be a sequence of iid random variables with arbitrary marginal density f_V, and independent of U_0. The V_n are called innovations. Then the recursive scheme $U_n = \langle U_{n-1} + V_n,\, n > 0 \rangle$ is marginally uniform on $[0,1]$. Note, that the distribution of the V_n is completely irrelevant!
4. Foreground/Background Schemes: TES sequences consist of a background sequence $\{U_n\}_{n=0}^{\infty}$ of marginally uniform distributed random variables constructed as described in 3, by using appropriate innovations V_n. The inversion method is then used to transform this sequence into a sequence $\{X_n\}_{n=0}^{\infty}$ by using the empirically observed histogram.

Creating TES Processes

The inversion method needs the empirical histogram. Unfortunately, the desired autocorrelation function cannot be modeled directly, but has to be searched for manually by using the TES workbench ([16]).

Possible Applications

TES models allow to include a variety of different autocorrelation functions, from slowly decaying, alternating in sign to oscillatory. Thus, TES models are well suited to model VBR coded video, Ethernet traffic and web traffic. In [17], a model for MPEG-1 traffic is given that splits the observed MPEG-1 traffic into two traffic streams.:

1. Slow time scale traffic: MPEG-1 traffic consists of I, B and P pictures. The used encoder in [17] produces a sequence of IBBPBBPBBPBB sequences, called Group of Pictures (GOP). Over 8 such GOPs, the I, B and P sizes are averaged independently. The slow time scale traffic then consists of 8 IBBPBBPBBPBB sequences, where all I, B and P pictures have the same value.
2. Fast time scale traffic: This is the difference of the slow time scale traffic to the observed traffic.

The four random processes (I, B, P, fast time scale) then were modeled by using TES processes.

VIII. Self-Similar Traffic Models

Empirical measurements of traffic have often shown the property of self-similarity, at least, if the traffic is high. A zero-mean, stationary time series $X = \{X_n\}_{n=0}^{\infty}$ can be m-aggregated to $X^{(m)} = \left\{X_k^{(m)}\right\}_{k=0}^{\infty}$ by summing the original time series over non-overlapping blocks of size m. Then, X is said to be H-self-similar if for all positive m, $X^{(m)}$ has the same distribution as X, rescaled by m

$$X_n = m^{-H} \sum_{i=(n-1)m+1}^{nm} X_i = m^{-H}\, X^{(m)}$$

for all $m \in N$. Self-similarity can be described by the Hurst parameter H, for which $0.5 \leq H \leq 1$ holds, $H = 0.5$ indicating no self-similarity and $H = 1$ indicating perfect self-similarity. If the equality holds only for variances and autocorrelation function, the process is called second order self-similar. There are several methods for estimating H from an empirical time series:

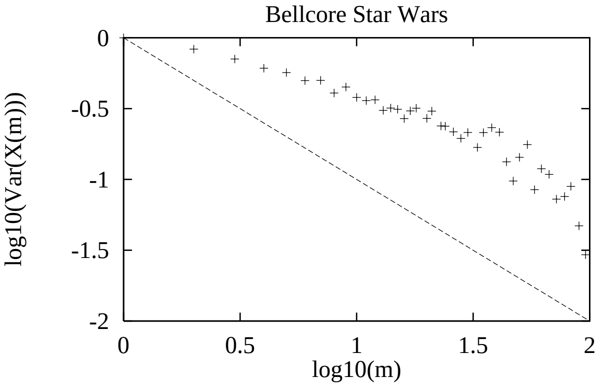


Fig. 1. Variance-Time plot of Bellcore Star Wars file.

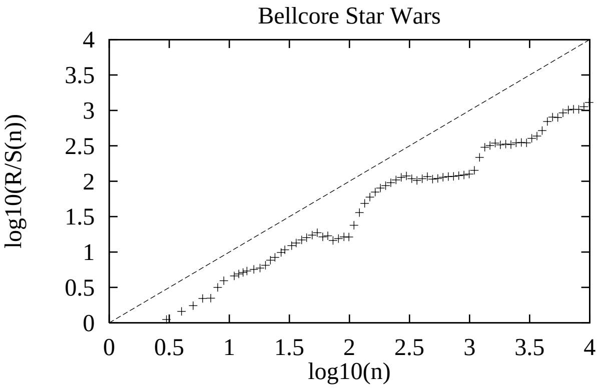


Fig. 2. R/S statistic plot of Bellcore Star Wars file.

1. Variance-Time plot ([2],[18]): plots $var(X^{(m)}/m)/var(X)$ against m on a log-log scale. A straight line with slope $(-\beta)$ greater than -1 indicates self-similarity with $H = 1 - \beta/2$. Figure 1 shows a Variance-Time plot of Mark Garrett's Star Wars video trace file ([19]) and a reference line $f(x) = -x$. In this file, 12 I, B and P frames of the MPEG-1 encoded Star Wars movie yield one Group Of Pictures (GOP), the plot showing the Variance-Time plot for the GOP time series. The estimated slope in this case is -0.495, yielding a Hurst parameter H of 0.75 (ref. [24]).
2. R/S plot ([2],[1]): The rescaled range statistic R/S grows like a power law with exponent H for self-similar traffic. Figure 2 shows a R/S statistic plot of the same vbr trace file ([19]) and the reference line $f(x) = x$). The estimated slope in this case is $H = 0.84$ (ref. [24]).
3. Periodogram method ([2],[18]): The shape of the power spectrum of a self-similar time series is a straight line on a log-log plot with slope $\beta - 1 = 1 - 2H$ close to the origin.
4. Whittle estimator: provides a confidence interval for H ([2]). In [20], an S-PLUS program for the calculation of the Whittle estimator is given. Before estimating, however, an appropriate stochastic model has to be chosen.
5. Correlogram plot: plotting the autocorrelation function $\rho(k)$ against the lag k on a log-log-scale yields another estimate for H ([20]). For a self-similar time series, such a plot will have a slope of $2H - 2$. In Figure 3, this is done again for the Star Wars GOP series. The slope is estimated to be -0.29, yielding $H = 0.86$.

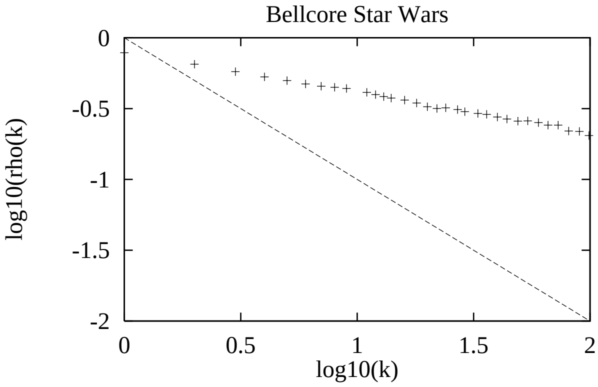


Fig. 3. Correlogram plot of Bellcore Star Wars file.

6. Wavelet estimator ([21],[22]): In wavelet analysis the signal $x(t)$ is analyzed by using a set of orthogonal basis functions $\phi_l^m(t)$, called wavelet functions, yielding the coefficients d_j^m (ref. to 3.7.4). Estimating $\widehat{H}(j_1, j_2)$ for appropriate scalings j_1 and j_2 is done by plotting

$$\log_2\left(\widehat{\Gamma}\left(2^{-j}v_0\right)\right) = \log_2\left(\frac{1}{n_j}\sum_m \left|d_j^m\right|^2\right)$$

against j, and applying linear regression. Here, $n_j = 2^{-j}n$ and v_0 is an appropriately chosen reference frequency. A confidence interval for H is given by

$$\widehat{H} - \sigma_{\widehat{H}} z_\beta \leq H \leq \widehat{H} + \sigma_{\widehat{H}} z_\beta,$$

where z_β is the quantile of the standard Gaussian distribution and

$$\sigma_{\widehat{H}}^2 = var\,\widehat{H}(j_1, j_2) =$$

$$= \frac{2}{n_{j_1}\ln^2 2}\frac{1-2^j}{1-2^{-(J+1)}(J^2+4)+2^{-2J}}.$$

Self-similarity has strong influence on the resulting traffic and has the following properties:

- Long range dependencies: The autocorrelation function decays like a power law rather than exponentially.
- Heavy tailed distributions (like Pareto distribution) with infinite variance are observed. Extremely large values are more likely. The tail of a distribution is said to be heavy tailed, if it decays like a power law: $P\{X > x\} = 1 - F(x) = \widehat{F}(x) \sim x^{-\alpha}$. There are several methods to estimate the tail index α from given data:
  1. Plotting $\widehat{F}(x)$ on log-log-axes ([23]): Plotted in this way, heavy-tailed distributions have the property that $\frac{d\log\widehat{F}(x)}{d\log x} \sim -\alpha$, for large x. Linear behavior in the upper tail gives evidence of a heavy-tailed distribution.

2. The Hill-estimator ([23]): The Hill estimator gives an estimate of α as a function of the k largest elements in the data set:

$$H_{k,n} = \left(\frac{1}{k} \sum_{i=0}^{k-1} \left(\log X_{(n-i)} - \log X_{(n-k)} \right) \right)^{-1}.$$

- Traffic bursts are observed. Such bursts in contrast to Poisson arrival processes with the same mean arrival rate will increase the mean waiting time and cell-loss probability due to buffer overflow drastically. Traffic bursts generally describe the ability of the process to stay below or above the average for a long time, and are strongly tied to large positive autocorrelations. There are some popular indices for burstiness ([1]):

1. Peak-To-Mean ratio (PMR).
2. Coefficient of variation for inter-arrival times.
3. Hurst parameter (according to self-similarity) H.
4. Poisson traffic comparison (PTC).
5. Infinite server effect (ISE).
6. Index of dispersion for intervals (IDI).
7. Index of dispersion for counts (IDC).
8. The peakedness functional .

Self-similar traffic has been observed in Ethernet ([2]) and ATM traffic, Telnet and FTP traffic ([3]), web traffic ([18]) and VBR-video traffic ([24]). The following sections will show some self-similar stochastic processes.

A. Fractional Brownian Motion

The zero mean Gaussian process $B_H(t)$ with Hurst parameter H is defined by

1. $E[B_H(t)] = 0$.
2. $B_H(0) = 0$.
3. $B_H(t+\delta) - B_H(t)$ is normally distributed $N\left(0, \sigma |\delta|^H\right)$.
4. $B_H(t)$ has independent increments.
5. $E[B_H(t) B_H(s)] = \sigma^2/2 \left(|t|^{2H} + |s|^{2H} - |t-s|^{2H}\right)$.

$B_H(t)$ is exactly self-similar, perfectly determined by H.

In [25], FBm is defined to characterize the number of arrivals in the interval $(0,t)$:

$$N_t = mt + \sqrt{am} Z_t,$$

where m denotes the mean of the process, a is the coefficient of variation $var[T]/E[T]$, and Z_t is the normalized FBm with Hurst parameter H.

Creating FBm traffic

Fractional Brownian Motion (FBm) ([26]) can be created, for example, by the Random Midpoint Displacement (RMD) method ([27])]:

1. Start with two end-points
2. Add one point in the middle of these two points, and displace it with a random term (which depends on H).
3. Add points between all existing points and displace them with random terms, until the desired number of points has been generated.

In [28],[29], FBm is created by using wavelets.

Possible Applications

FBm can be used to model the sum or integral of self-similar traffic (as observed in network buffers, file sizes of audio/video streams, ...). Its increments/derivative can yield the self-similar fractional Gaussian Noise.

B. Fractional Gaussian Noise

The increments of FBm are known as Fractional Gaussian noise (FGn) ([26]) and form a stationary process $G_H(t)$ with the following properties:

1. $G_H(t) = \frac{1}{\delta}(B_H(t+\delta) - B_H(t))$.
2. $G_H(t)$ is normally distributed $N\left(0, \sigma |\delta|^{H-1}\right)$.
3. $E[G_H(t+\tau) G_H(t)] = \sigma^2 H (2H-1) |\tau|^{2H-2}$ for $\tau \gg \delta$.

Discrete time FGn also has the following autocorrelation function ([20],[1]):

$$\rho_X(k) = \frac{1}{2}\left(|k+1|^{2H} - 2|k|^{2H} + |k-1|^{2H}\right), \; k \geq 1.$$

It can be necessary to truncate the FGn series, as negative values are possible.

Creating FGn traffic

In [30], an algorithm is given to efficiently create estimated discrete-time FGn. The Algorithm first generates an estimate of the power series $f(\lambda, H)$ of the desired traffic stream at the discrete frequencies $\lambda_j = 2\pi j/n$, $j = 1, \ldots, n/2$. Here, only the Hurst parameter H is necessary (for estimation see above). After some transformations, a sequence of n complex numbers is obtained, which is transformed back via the inverse Fourier transformation to obtain a sequence $\{x_k\}_{k=1}^n$. An instance of the algorithm is explicitly stated, programmed in the statistics language S.

In [20], an S-PLUS program creating FGn is given. Here, the series is generated by using the according covariances up to lag n and applying inverse Fourier transformation.

In [31], an algorithm initially proposed in [32] is briefly described.

Possible Applications

FGn are exactly second-order self-similar and are thus good candidates in modeling self-similar traffic:

- Ethernet ([2])
- ATM
- VBR coded video
- Web traffic, cache
- Telnet, FTP

C. ARFIMA

Fractional ARIMA models (ARFIMA or FARIMA) ([2],[20]) are built on classical ARIMA models. $\{X_n\}_{n=0}^{\infty}$ is called an ARFIMA(p,d,q) process, if $\left\{\triangle^d X_n\right\}_{n=0}^{\infty}$ is an ARMA(p,q) process for some non-integer $d > 0$. B is the Backshift-operator $B(X_n) = X_{n-1}$ and $\triangle^d$ can be represented by

$$\triangle^d = (1-B)^d = \sum_{u=0}^{\infty} \pi_u B^u$$

with $\pi_0 = 0$ and

$$\pi_u = \frac{\Gamma(u-d)}{\Gamma(u+1)\Gamma(-d)} = \prod_{k=1}^{\infty} \frac{k-1-d}{k},\ u = 1, 2, \ldots$$

Note that using the gamma function Γ is the natural generalization for d being an integer, because in that case, $(1-B)^d$ is a finite sum and the coefficients π_u are binomial coefficients.

ARFIMA processes are asymptotically self-similar, if $0 < d < 0.5$, with Hurst parameter $H = d + 0.5$. For large lags, the correlations of an ARFIMA(p,d,q) process are similar to those of an ARFIMA(0,d,0) with the same d.

Creating ARFIMA models

The fractional differentiating parameter d can be estimated from a previous estimate of the Hurst parameter H by using the above equation. After this, the observed time series must be fractionally differenced to yield a new time series

$$\left\{ Y_n = (1-B)^d X_n \right\}_{n=0}^{\infty}.$$

For the new time series, an appropriate ARMA(p,q) model is then created ([12]).

In [24], an algorithm is given for the creation of ARFIMA(0,d,0) processes with arbitrary marginal distributions. The algorithm, though, is of complexity and required 10 hours of CPU time for generating 171,000 points on an 1994 state of the art workstation.

In [20], an S-PLUS program for generating ARFIMA(0,d,0) series is given.

Possible Applications

ARFIMA models are similar to FGn, yet they are very flexible due to the natural correspondence to ARIMA(p,d,q) models and to their higher number of parameters.

In [24], VBR coded video traffic is modeled with ARFIMA models.

D. Wavelets

The above described stochastic models try to capture short- and long-term dependencies as observed in VBR video or Ethernet traffic. Wavelets now provide a means of transforming the original self-similar process into a new process with much less self-similar behavior. For this new process, simpler models can be applied. Traffic is then generated first in the wavelet domain, and then transformed back into the time domain by applying the inverse wavelet transformation ([14],[28],[29]).

Like in the Fourier transform, the observed values $\{X(t)\}_{t=0}^{2^K}$(for some integer K) of an equally spaced, discrete-time process are analyzed according to a complete orthonormal basis of the Hilbert space $L^2(R)$ of all squared integrable functions ([33]). The members ϕ_j^m of this orthonormal basis are derived from a special function $\phi(t)$, the mother wavelet, by translation in the time domain, and scaling in the frequency domain ([14]):

$$\phi_j^m(t) = 2^{-j/2}\phi\left(2^{-j}t - m\right).$$

Here, the positive integer m denotes the translation index, while the positive integer j denotes the scaling index. The task of wavelet transformation is to find wavelet coefficients d_j^m such that

$$x(t) = \sum_{j=0}^{K} \sum_{m=0}^{2^{K-j}-1} d_j^m \phi_j^m(t) + \phi_0$$

holds for $0 \leq t < 2^K$. This is called the inverse wavelet transform. The wavelet coefficients are given by

$$d_j^m = \sum_{t=0}^{2^K-1} x(t)\,\phi_j^m(t).$$

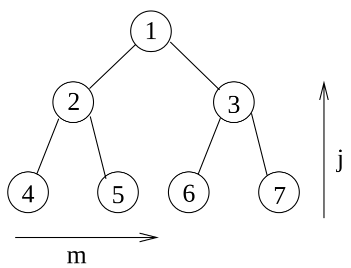
There are several popular mother wavelets. One, for example, is the Haar wavelet

$$\phi(t) = \begin{cases} 1, & if\, 0 \leq t < 1/2 \\ -1, & if\, 1/2 \leq t < 1 \\ 0, & otherwise \end{cases}.$$

The corresponding Haar wavelets $\phi_j^m(t)$ are scaled and shifted versions of $\phi(t)$. For Haar wavelets, the corresponding wavelet coefficients are given by

$$d_j^m = 2^{-j/2}\left(\sum_{t=m2^j}^{(m+0.5)2^j-1} x(t) - \sum_{t=(m+0.5)2^j}^{(m+1)2^j-1} x(t) \right).$$

Though the wavelet coefficients have two indices, they can be transformed into a discrete-time process $\{d_s\}$ by using a triangular scheme:



Furthermore, if the observed process consists of random variables, then the wavelet coefficients themselves are also random variables. Due to the one-to-one correspondence of the input process and wavelet coefficient process, the statistical properties of $x(t)$ are completely determined by the statistical properties of the wavelet coefficients.

Experiments show that the auto-correlation function of the new discrete-time process decays much faster (exponentially) than that of the original (possibly self-similar) process. Thus, simpler models like Gaussian type models can be used to model this new process.

Creating Network Traffic with Wavelets

Network traffic is generated in the following way ([14]):

1. Sample $N = 2^K$ observation values.
2. Compute the wavelet coefficients d_j^m for this data.
3. Transform the indices to get the new process d_s.
4. Model d_s by a simple Gaussian type, stochastic process (n-th order Markov).
5. Create 2^K variates in the wavelet domain by using this stochastic process.
6. Create the required $x(t)$ in the time domain by applying the inverse wavelet transform.

As the complexity of the wavelet transform and inverse wavelet transform is of order $O(N)$, where $N = 2^K$ is the length of the time series, the complexity of the whole algorithm is $O(N)$. This makes the scheme very efficient!

Possible Applications

Wavelets are capable of capturing both short-range and long-range dependencies ([14]). They are thus well suited for modeling Ethernet, ATM, VBR, Telnet, FTP and web traffic.

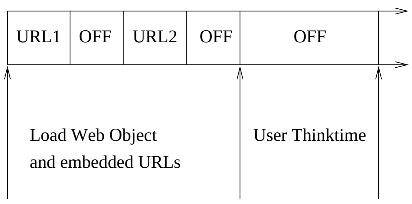
E. On/Off Processes

A large number of superimposed heavy tailed On/Off processes ([34]) can yield self-similar traffic as well. An On/Off process is either in state On or Off. We construct a time series by observing the number of On-processes at any time point. If On-times and Off-times are drawn from a heavy tailed distribution like the Pareto distribution with parameters α_1 and α_2, then the observed stochastic process is a self-similar fractional Gaussian noise process with $H = (3 - \min(\alpha_1, \alpha_2))$.

Creating Traffic with On/Off Processes

On/Off processes are mapped to network traffic in the following way:

- Each process corresponds to a workstation either being silent (Off) or sending data at a constant rate (On).



- Each process corresponds to a web user, On-times are given by the web document transmission times and Off-times are the time intervals between the transmissions ([35]). This model can be refined by modeling active-Off times (time between the transmission of two files belonging to the same HTML document) and inactive-Off times (time between user actions) as well. Transmission times of files are a function of their length, thus the distribution of web file length has been shown to be heavy-tailed. Zipf's law connects this file length to the number of times, a file has been transmitted (file popularity).

In [36], mixtures of fractal On/Off processes, called fractal point processes, are discussed.

Possible Applications

On/Off processes can be used to create network traffic at the packet level, or streams of requests at a higher level, like transferring files over the net ([35]).

F. Poisson-Zeta Process

A Poisson-Zeta process $PZ[\alpha, \rho]$ is a discrete time On/Off process, where the number of bursts at each time point n is given by a Poisson distribution with mean α. Each burst generates one cell (ATM) per time unit during its duration, the duration l of each burst has independent identical Zeta distributions $\{g_h\}_{h=1}^{\infty}$ (like a discrete version of Pareto) with parameter $1 < \rho < 2$. g_h is the probability that the burst will last for h time units. In [37] it has been demonstrated that this process is asymptotically self-similar.

Possible Applications

In [37], an ATM switch has been fed with a Poisson-Zeta process.

G. Deterministic Chaotic Maps

Deterministic chaotic maps are related to On/Off sources ([38]). Here, the driving sequence is derived from chaotic processes having the SIC (Sensitive dependence on Initial Conditions) property. In such processes, the observed trajectories severely depend on the starting point. Changes of these starting points have exponential effects on the observed trajectories. Traffic is produced by creating the stochastic processes x_n and y_n:

$$\begin{array}{lll} x_{n+1} = f_1(x_n), & y_n = 0, & if\, 0 < x_n \le d \\ x_{n+1} = f_2(x_n), & y_n = 1, & if\, d < x_n < 1 \end{array}$$

for an appropriately chosen d and map functions $f_1(x)$ and $f_2(x)$. If x_n is above the threshold, then one traffic packet is generated. In [38], two maps, the piecewise linear map

$$x_{n+1} = \begin{cases} \frac{x_n}{(1-\lambda)}, & 0 < x_n \le (1-\lambda) \\ \frac{x_n-(1-\lambda)}{\lambda}, & (1-\lambda) < x_n < 1 \end{cases}$$

and the intermittency map

$$x_{n+1} = \begin{cases} \varepsilon + x_n + c x_n^m, & 0 < x_n \le d \\ \frac{x_n - d}{1-d}, & d < x_n < 1 \end{cases}$$

where

$$c = \frac{1-\varepsilon-d}{d^m}$$

are then defined.

Possible Applications

In [38], Chaotic Maps are proposed as models for generating packet network traffic.

H. Self-Similarity Through Aggregation

A more sophisticated process, yielding self-similar traffic through aggregation, is given in [2] and [1]. Let $\{I_K\}_{k=0}^{\infty}$ be a sequence of iid integer-valued random variables with asymptotic tail probabilities obeying the power law (for example Pareto)

$$P\{I_k \geq t\} \approx t^{-\alpha} h(t), \; as\, t \to \infty,$$

where $1 < \alpha < 2$ and $h(t)$ is a slowly varying function. Let $\{G_k\}_{k=0}^{\infty}$ be an iid sequence, independent of $\{I_k\}$, with $E[G_k] = 0$ and $E[G_k^2] < \infty$. Define the stationary sequence

$$S_k = S_0 + \sum_{j=1}^{k} I_j, \; k \geq 1,$$

with an appropriately chosen S_0. Then define $W = \{W_k\}_{k=1}^{\infty}$:

$$W_k = \sum_{n=1}^{k} G_n 1_{(S_{n-1}, S_n]}(k).$$

Construct M iid copies $W^{(1)}, \ldots, W^{(M)}$ of W. Then the process

$$W^* = \{W_k^*(M)\}_{k=0}^{\infty},$$

given by

$$W_k^*(M) = \begin{cases} 0, & k = 0 \\ \sum_{n=1}^{k} \sum_{m=1}^{M} W_n^{(m)}, & k > 0 \end{cases}$$

behaves like FBm, provided that k and M are large and $k \ll M$.

Possible Applications

Any kind of network traffic showing self-similar behavior.

I. The $M/G/\infty$ Model

In [3], an $M/G/\infty$ model is stated, which is capable of constructing asymptotically self-similar traffic. Let $\{X_t\}_{t=0,1,2,\ldots}$ be the counting process denoting the number of customers in the $M/G/\infty$ system at time t. If customers have a service distribution function F, then the autocorrelation function of X_t is

$$r(k) = \rho \int_k^{\infty} (1 - F(x))\, dx,$$

where ρ is the rate of the Poisson process of customers arriving at the system. If F is the Pareto distribution, then

$$r(k) = \rho \int_k^{\infty} \left(\frac{\alpha}{k}\right)^{\beta} dx = \frac{\rho \alpha^{\beta}}{\beta - 1} k^{(1-\beta)},$$

and thus the process is asymptotically self-similar.

Possible Applications

In [3], various aspects of Telnet and FTP traffic in connection with $M/G/\infty$ models are discussed.

J. Superimposing AR(1) Processes

In [2] it is stated that when aggregating many simple AR(1) processes, where the AR(1) parameters are chosen from a beta-distribution on $[0, 1]$ with shape parameters p and q, then the superposition process is asymptotically self-similar. Also, the Hurst parameter H depends linearly on the shape parameter q of the beta-distribution. Obviously, creating the AR(1) processes can be done in parallel.

Possible Applications

In[15], the mixing of two AR(1) processes is used to generate ATM traffic.

K. Self-Similar Markov Modulated

In [39], self-similarity is simulated by using a Markov modulated discrete-time, discrete-state process. The proposed modulating Markov chain depends only on 3 parameters.

Possible Applications

VBR, Telnet, FTP, Ethernet, Web, etc. are possible applications.

L. The GBAR and GBMA Processes

The GBAR process [40] is a Gamma-Beta autoregressive process. Let $Z_{i-1} \sim Gam(\alpha, 1)$, $W_i \sim Gam((1-\rho)\alpha, 1)$, and $B_i \sim Beta(\alpha\rho, \alpha(1-\rho))$ be independent, then

$$Z_i = B_i Z_{i-1} + W_i$$

is also $Gam(\alpha, 1)$-distributed. The autocorrelation function of this process is geometric. As is stated in [41], triangular shaped autocorrelation functions can be derived from moving averages of Gamma processes. Any kind of autocorrelation can be modeled by weighting Gamma processes with Beta distributions, then applying the moving average filter to it (GBMA process).

Possible Applications

In [41], the GBAR and GBMA models are used to model the sizes of MPEG frames. Other applications include Ethernet traffic, Web, WAN, etc.

M. Spatial Renewal Processes

Spatial renewal processes ([17],[42]) consist of two background processes, the first being a point process $T = \{T_0 \leq 0, T_n, n \geq 1\}$, such that the inter-renewal times $T_n - T_{n-1}$, $n \geq 1$ are iid with distribution function $F_T(t)$. A second process $\{X_n\}_{n=0}^{\infty}$ consists of iid random variables with the desired marginal distribution as observed. These two processes together then yield the foreground process $Y_t = X_n$ for $T_n \leq t < T_{n+1}$. If the desired autocorrelation function $\rho(t)$ for $\{Y_t\}$ is either given empirically or known analytically (for example, if we want to generate FGn, then the autocorrelation function is known for discrete points and must be extended to the set of real numbers), then we just have to construct $F_T(t)$ over equation

$$1 - \rho(t) = \mu^{-1} \int_0^t (1 - F_t(u))\, du, \; t \geq 0,$$

or equivalently

$$-\frac{d}{dt}\rho(t) = \mu^{-1}(1 - F_T(t)),\ \rho(0) = 1,\ t \geq 0,$$

where

$$\mu = \int_0^\infty (1 - F_T(u))\ du.$$

In order to yield valid distribution functions, the used autocorrelation function must be a decreasing, concave-up function. The constructed foreground process $\{Y_t\}$ will then have the desired marginal distribution and the required autocorrelation function! In [42], the distribution function for FGn is stated explicitly.

Possible Applications

In [17], SRP are used to model MPEG-1 encoded video streams. Other possible applications include Ethernet and ATM traffic, WAN and Web traffic.

N. Multifractal Traffic

In [43], the multifractal nature of WAN traffic is demonstrated. In contrast to monofractal (self-similar) traffic, where the local scaling behavior is constant, multifractal traffic takes into account the changing local scaling behavior over time. This local scaling behavior is measured as the rate, at which the number of bytes/packets observed in the interval $[t_0, t_0 + \delta t]$ tends to zero as $\delta t \to 0$. In [43], this local scaling behavior is calculated by using wavelet transforms. The multifractal property is then motivated by the cascading nature of WAN traffic (each trace consists of sessions, each session consists of traffic requests, each traffic request consists of TCP connections, each TCP connection consists of IP packets, ...).

IX. Overview of Traffic Generators

In this section, an overview of the bibliography for modeling and generating traffic of certain types is given. In order to provide some starting point, in the papers below either explicit models for the respective traffic types have been proposed, or the authors themselves have proposed to use their models for these types.

A strict distinction between these traffic types, however, is not always feasible, as transporting multimedia traffic will be a dominant factor in tomorrows networks, and, for example, VBR encoded video will be transported over ATM, Ethernet and subsequently as WAN traffic over the Internet. Also, multimedia traffic will be an important part of future web traffic. Thus, models for one type of traffic are often applicable to all other traffic types as well.

A. VBR Video Traffic / MPEG

VBR encoded video traffic is by its very nature bursty and shows strong correlations between successive frames sizes ([24]). Popular codecs include MPEG-1, MPEG-2 and MPEG-4 for video encoding, and H.261 and more recently H.263 for video conferencing. These codecs use DCT transformations to reduce spatial redundancy and prediction and motion compensation to reduce temporal redundancy. The video stream is sent in sequences of frames of types I, P, B, and PB (two pictures coded as one frame).

Burstiness is introduced by sudden scene changes, shifting the average frame sizes away from the mean. Inside scenes, prediction and motion compensation keep frames of the same type (I, P, B, PB) from varying too heavily in size. Thus, either frames of the same type or the sums of the sizes of frames belonging to one Group of Picture (GOP) show strong correlations.

- [44]: Geometric On/Off.
- [45]: Periodic Markov modulated batch Bernoulli.
- [5]: Markov chain.
- [41]: GBAR, GBMA.
- [24]: ARFIMA(0,d,0).
- [46]: Discrete AR, Markov chain, Scene changes.
- [47]: Markov chain.
- [31]: FGn, Arbitrary marginal distribution.
- [48]: TES.
- [17]: Multiple time scale TES, Spatial renewal processes.
- [49]: FBm, DAR (Markov), Markov chain.
- [50]: TES.
- [51]: Generalized TES.
- [52]: Leaky bucket, empirical envelopes.
- [53]: Markov modulated On/Off.
- [54]: TES.
- [30]: FGn.
- [55]: Markov chains.
- [13]: DAR(p).
- [56]: TES.
- [14]: Wavelets.
- [42]: Spatial renewal processes.

B. Ethernet Traffic

In [2] it has been demonstrated that Ethernet traffic is bursty and highly self-similar. One explanation for this is given by the theory of On/Off processes. If each workstation is regarded to be either in state On (sending data) or Off (doing nothing), then the superposition of such On/Off sources can yield asymptotically self-similar traffic ([34]).

- [21]: Wavelets.
- [38]: Chaotic Maps.
- [2]: FGn, ARFIMA(p,d,q), aggregation.
- [39]: self-similar Markov modulated.
- [36]: Fractal point processes.
- [14]: Wavelets.

C. ATM

ATM will be dominated by tomorrows audio and video traffic. Thus, the models below are often similar to models for VBR encoded video.

- [15]: Superimposing two AR(1) processes.
- [57]: On/Off sources.
- [58]: heavy-tailed Renewal.
- [37]: Poisson-Zeta (On/Off).
- [59]: On/Off sources.

- [60]: FBm, mix of two AR(1), M/Pareto.
- [61]: On/Off sources.
- [62]: MMBP.

D. Wan, TCP, Telnet, FTP

Measurements of WAN traffic often include traffic generated by Telnet or FTP downloads ([63], [3]). Again, burstiness and self-similarity is observed. In [43], this is explained by the inherent hierarchical nature of WAN traffic.

- [8]: B-MAPs.
- [22]: Wavelets, $M/G/\infty$.
- [43]: Multifractals
- [25]: Poisson, MMPP, AR(1), Weibull, Pareto, FBm.
- [63]: Inter-arrival times, autocorrelation function.
- [3]: Pareto, $M/G/\infty$, Log-normal.
- [36]: Fractal point processes.

E. Web Traffic

Web based network traffic also shows self-similar behavior. In [34] this is explained by the fact that the file transfers can be seen as On/Off processes, where the On time, given by the sizes of transferred files, is drawn from heavy-tailed distributions. The aggregate of such traffic is then asymptotically self-similar.

- [35]: On/Off sources, Zipf's Law.
- [34], [18]: On/Off sources, Zipf's law.
- [64]: Poisson.
- [22]: Wavelets, $M/G/\infty$.
- [65]: ARIMA(p,d,q).

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